

1. Battery Life Testing

A company tested the lifespan of 10 randomly selected batteries, recording an average time to failure of 9.5 hours with a standard deviation of 1.4 hours.

a) Determine a 95% confidence interval for the population mean time to failure.

$n=10$
 $\bar{x} = 9.5$
 $\alpha = 1 - 0.95 = 0.05$
 $\alpha/2 = 0.025$
 $s = 1.4$

$\Rightarrow \left[\bar{x} \pm t_{\alpha/2, (n-1)} \frac{s}{\sqrt{n}} \right]$
 $\Rightarrow \left[9.5 - d, 9.5 + d \right]$

$t_{\alpha/2, (n-1)} \cdot \frac{s}{\sqrt{n}} = \frac{2.262 \times 1.4}{\sqrt{10}} = d$

b) Determine a 99% confidence interval for the population mean time to failure.

$\alpha = 1 - 0.99 = 0.01$
 $\alpha/2 = 0.005$

$t_{0.005, 9} = 3.250$
 $\Rightarrow 3.250 \times \frac{1.4}{\sqrt{10}} = d$

$[9.5 - d, 9.5 + d]$

99%

2. Water Bottle Volume Testing

A water bottle company checks that each bottle holds approximately 500 ml of water. A random sample of 12 bottles yields the following volumes (in ml): 498, 502, 499, 500, 497, 504, 501, 498, 500, 503, 497, 496.

$\Rightarrow n=12, \bar{x} = 499.5, s = 2.539$

a) Determine a 95% confidence interval for the population mean bottle volume.

$\alpha = 0.05$
 $\alpha/2 = 0.025$
 $t_{\alpha/2, n-1} = 2.201$

$d = t_{\alpha/2, n-1} \times \frac{s}{\sqrt{n}} = 1.6132$

$[497.89, 501.11]$

$[0.025, 11]$

$\text{mean} = 499.5$

b) Determine a 98% confidence interval for the population mean bottle volume.

$\alpha = 0.02$
 $\alpha/2 = 0.01$

$d = t_{\alpha/2, n-1} \times \frac{s}{\sqrt{n}}$

$t_{0.01, 11} =$

$[\bar{x} - d, \bar{x} + d]$

1. Heart Rate Measurement

A cardiologist records the resting heart rate of 300 randomly selected male patients aged 20-30. The sample shows an average heart rate of 72.3 beats per minute, with a standard deviation of 5.1.

- a) Determine a 95% confidence interval for the average heart rate in this age group.

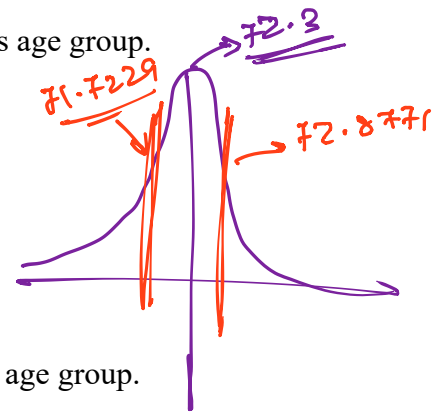
$$\bar{x} = 72.3$$

$$s = 5.1$$

$$\alpha/2 = 0.025$$

$$d = z_{\alpha/2} \times \frac{s}{\sqrt{n}} = 0.577$$

$$[71.7229, 72.8771]$$



- b) Determine a 99% confidence interval for the average heart rate in this age group.

$$\alpha/2 = 0.005$$

$$d = z_{0.005} \times \frac{s}{\sqrt{n}} = 0.7584$$

$$[72.3 - 0.7684, 72.3 + 0.7584]$$

2. Salary Survey

A survey aims to estimate the average annual salary of employees in a city. A random sample of 500 workers reveals a mean salary of \$42,350 with a standard deviation of \$5,300.

- a) Determine a 95% confidence interval for the average salary of workers in this city.

- b) Determine a 98% confidence interval for the average salary of workers in this city.

1. Machine Part Variation

A manufacturer measures the diameter of 15 randomly selected parts to estimate the variability in the production process. The sample data yields a mean diameter of 5.2 cm and a standard deviation of 0.3 cm.

- a) Determine a 98% confidence interval for the population standard deviation of the part diameter.

- b) Determine a 95% confidence interval for the population standard deviation of the part diameter.

2. Quality Control on Bottle Caps

A company inspects 18 randomly selected bottle caps and calculates the mean and standard deviation of their widths as 2.4 cm and 0.2 cm, respectively.

- a) Determine a 99% confidence interval for the population standard deviation of the cap width.

- b) Determine a 95% confidence interval for the population standard deviation of the cap width.

3. Customer Satisfaction Survey

A company conducts a customer satisfaction survey for its new product. Out of 1,200 surveyed customers, 780 report that they are satisfied with the product.

a) Determine a 90% confidence interval for the true proportion of satisfied customers.

b) Determine a 95% confidence interval for the true proportion of satisfied customers.

4. Confidence Intervals for Proportions with Various Sample Sizes

For each of the following sample sizes, determine a 95% confidence interval for the population proportion if the observed proportion (\hat{p}) is 0.75.

a) $n = 500$

b) $n = 750$

c) $n = 1000$

d) $n = 1500$

e) $n = 2000$

